

2 · 項目簡介

(項目所屬科學技術領域、主要研究內容、發現點、科學價值、同行引用及評價等內容。)

This project concerns with computational mathematics. More specifically, we study efficient methods for solving Toeplitz systems.

Toeplitz systems originate from function theory. They were proposed by Toeplitz around 1910. These systems were studied by many great mathematicians such as Hilbert, Caratheodory, Fischer, Frobenius, Riesz, Schur and Szego. Toeplitz systems arise in a variety of applications, for instance, numerical differential equations and integral equations; data and image processing; queuing and control theory; financial mathematics and statistical theory. The topic studied in this project, namely preconditioning techniques for Toeplitz systems, has great scientific value. Moreover, it is a very active topic and has been in the frontier of computational mathematics for decades.

Solving Toeplitz systems by preconditioned conjugate gradient (PCG) method with circulant preconditioners was proposed and studied by G. Strang and R. Chan in 1989. In their paper [Toeplitz equations by conjugate gradients with circulant preconditioner, SIAM J. Sci. Stat. Comput., Vol. 10, pp. 104--119], they showed, for the first time, that PCG method with circulant preconditioners can be used to solve Toeplitz system efficiently. To be more precise, they proved that the computational cost for solving an order n Toeplitz system only needs $O(n \log(n))$ operations which gives the least computational cost among all methods developed so far for solving Toeplitz systems. The idea and technique in that paper established the framework for the study in this area.

Xiao-Qing Jin, the team leader of this project, initiated the first study on fast algorithm for solving block Toeplitz systems during his PhD study under the supervision of R. Chan and he extended the method to many practical applications. As generally accepted by researchers in this area, when comparing with other existing methods for solving block Toeplitz systems, our method has the advantages that it involves the least computational cost and covers the broadest range of applications.

We summarize below the main discoveries obtained in this project. The ideas used are innovative and the results obtained are original. Our works in this area play leading roles on international level.

1. We discovered fast iterative method for block Toeplitz systems. In addition, the framework of the convergence analysis for such systems are established.
2. We found many applications of this method to differential and integral equations.
3. We constructed preconditioners for constrained and weighted least squares problems with Toeplitz block structure.

4. We applied our theory to some practical problems such as image processing and financial mathematics.

5. We proved some mathematical properties of preconditioned matrices for different kinds of circulant preconditioners.

Appraisal by Third Parties

1. "it (Developments and Applications of Block Toeplitz Iterative Solvers) should be of benefit to anybody with research interests in block Toeplitz systems." by Prof. Yimin Wei of Fudan University in The Bulletin of the International Linear Algebra Society, Image, Issue Number 31.

2. "The book (Developments and Applications of Block Toeplitz Iterative Solvers) is an excellent guide for beginners. Moreover, it is very interesting for specialists working in the field of Toeplitz matrices." by Prof. Daniel Potts in MathSciNet.

3. "the volume (An introduction to iterative Toeplitz solvers) is a very good introduction to modern preconditioners for the iterative solution of symmetric Toeplitz linear systems." by Prof. Daniel B. Szyld of Temple University in Mathematics of Computation, Volume 78, Number 226.

4. "this (An introduction to iterative Toeplitz solvers) is a very accessible textbook for anyone who wants a clear introduction to the problem." by Prof. A. Bultheel in MathSciNet.

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