2· 項目簡介

(*項目所屬科學技術領域、主要研究內容、發現點、科學價值、同行引用及評價等內容。*)

This project is on solving fractional partial differential equations (FPDEs) numerically in an efficient way. FPDEs are first discretized by using finite difference method which is proven to be consistent, unconditionally stable and convergent. Then fast algorithms are designed for solving those finite difference schemes with a significant speed up. Theoretical and practical supports are both established. FPDEs are recently useful in Physics, biology and finance. Since analytic solution is rare, therefore this project is significant.

Our first paper is on multigrid method for solving space fractional diffusion equations (SFDEs), published in a top 5% SCIE ranking journal, which broke the bottle neck on fast solution methods. Due to the nonlocal property of fractional derivatives, the coefficient matrix of finite difference schemes are dense so that typical methods required O(n^3) operation cost and O(n^2) memory where n is the number of grid points. The proposed method is based on the nearly Toeplitz structure of the coefficient matrix which solves the discrete system in O(n log n) operations with O(n) storage requirement only. Another important work is on circulant preconditioners for solving SFDEs, published in 2013 on the same journal. Based on the special matrix structure, by using fast Fourier transform, the proposed circulant preconditioned Krylov subspace method solves the discrete system with similar operation cost and storage, but with easier implementation. These two papers are always treated as a benchmark in the field. Afterwards, contour integral method and method based on exponential quadrature rule are proposed which further improve the efficiency.

Time FPDE is another challenging problem since the solution at certain time moment depends on all solutions in previous time. This makes the computational cost high when numerical solution for large time is desired. Standard time stepping method solve the problem in O(NM^2) operations, where N and M are the number of grid points in space and time, respectively. However, block triangular Toeplitz structure of the coefficient matrix is found so that under an epsilon circulant approximation to the triangular Toeplitz matrix, an efficient solution algorithm is developed with complexity O(NM log M). This is called the approximate inverse method and two more papers further improved the method for solving time FPDE with variable coefficients. The above works are published in top 10% SCIE journals.

High order finite difference scheme is an important part in the field since the higher the order of convergence, the faster the error tends to zero with respect to mesh size. In this project, some papers are devoted to high order method in which combined compact difference (CCD) method is the main technique. Based on the CCD method, sixth order in space and second order in time difference schemes are constructed for solving unsteady convection diffusion equations, and cubic nonlinear Schrodinger equations with stability analysis. The works are published in the rank 1 journal Computer Physics Communications. Some high order difference schemes are developed also for solving time fractional advection diffusion equations, distributed order differential equations, and time space FPDEs.

This project achieved a high reputation in the field. According to the web of science, all 19 papers in this project are published in SCIE journals in which 2 are in the ranked 1 journal, 7 are in top 10% journals except the 2 ranked 1 journal papers, 3 are in top 10 - 15% journals, 3 are in top 15 - 25% journals, 3 are in top 25 - 50% journals, and just 1 is out of 50%. The total SCIE citation of this project is 218 and our papers have been cited by famous researchers from Brown University, University of Oxford, Purdue University, Chinese Academy of Science, and Max Plank Institute.

（字數不超過 1200 字）