二· 項目簡介

（*項目所屬科學技術領域、主要研究內容、發現點、科學價值、同行引用及評價等內容。*）

The key concern of this project is on numerical methods for differential equations. Fractional differentiation and integration generalize the classic integer-order ones to arbitrary real-valued order. In recent decades, differential equations with fractional derivatives have attracted increasing attention due to its practical applications in viscoelastic models of fluid flow, underground water, financial mathematics and image processing etc. For example, for some applications in fluid dynamic, it is found that the wall-friction through the interface layer possesses a certain cumulative historical-dependence. Fractional derivatives can well capture this effect.

Since exact solutions to differential equations with fractional derivatives are usually not available, numerical methods for solving such equations are of great interest. However, due to the historical dependence of fractional derivatives, it is challenging to extend classical high-order methods to these equations. For example the L1 formula, which is a widely used discretization for the Caputo fractional derivatives, has truncation error of order depending on the order of the derivatives and is usually less than two.

The advantage of high order methods is that they can generate more accurate solutions than lower order ones under the same grid size. Improving the accuracy of discretization formulas for fractional derivatives and studying how these formula can be applied to different fractional differential equations are main concerns in this area. Seak-Weng Vong and Xiao-Qing Jin, the team members of this project, have done several important works along this directions. Inspired by our previous work on the weighted and shifted Grünwald formula for fractional derivatives, we developed several discretization formulas which can be used for linearized schemes to nonlinear differential equations with fractional derivatives. Furthermore, some efficient algorithms were proposed to reduce computational cost of the schemes. Our result has significant impact on the field (The representative paper [PUB21] is a highly cited paper).

In the following, we summarize main scientific discoveries in this project.

1. Theoretical analysis of high order difference schemes for linear equations with fractional derivatives: We construct schemes for differential equations with fractional derivatives and variable coefficients under different kinds of boundary conditions.
2. Generalization to nonlinear equations: The results are generalized to nonlinear equations by considering discretization in conservative form.
3. Linearized schemes using a weighted approach: When considering some nonlinear fractional differential equations, we proposed linearized schemes which can be solved without using iterative methods.
4. Study of non-smooth solutions with non-uniform girds: Initial singularities of solutions to some differential equations with fractional derivatives are handled accurately by using graded meshes.
5. Fast algorithms for improving efficiency: Several efficient approaches have been proposed to reduce computation cost of the approximate solutions.

In summary, among the 26 representative papers of the project, 16 of them are published in journals ranking above top 10% in the field. These results have been cited for 418 times and one of them is a highly cited paper.

Almost all results of the project are done by researchers in Macau or former students from University of Macau trained by the principle persons. The following students have been trained:
Pin Lyu (PhD), Dongdong Liu (PhD), Zhi Zhao (PhD), Zhibo Wang (PhD), Qing-Jiang Meng (PhD), Zhibo Wang (Msc), Pin Lyu (Msc), Chenyang Shi (Msc).
The students Pin Lyu (PhD) and Zhibo Wang (Msc) got the FDCT Award for Postgraduate in 2018 and 2014 respectively.

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