2· 項目簡介

(*項目所屬科學技術領域、主要研究內容、發現點、科學價值、同行引用及評價等內容。*)

Random Matrix Theory originated from multivariate statistics in the work of Hsu, Wishart and others in the 1930's and from nuclear physics in the work of Wigner (Nobel Laureate) in the 1950's. From 1960s to 1970s, through the fundamental work of Dyson, Mehta, des Cloizeaux, Widom and others, the subject evolved into a branch of mathematical physics. Its rapid development from 1990s stem from the work of Gross (Nobel Laureate) and Migdal, Douglas and Shanker on low-dimensional string theory, and the fundamental discovery of Tracy and Widom on the probability laws governing the largest and smallest eigenvalues of two families of Hermitian random matrices, the unitary Hermitian ensemble and the unitary Laguerre ensemble. Random matrix theory has become a very active area of research encompassing pure and applied mathematics; potential theory, operator theory, enumerative combinatorial problems, information theory to name a few.

A random matrix is a matrix whose elements are random variables such that under the eigenvalue-eigenvector decomposition, its eigenvalues maybe described by a joint PDF, essentially consists of the product of the absolutely value of the differences of the eigenvalues raise to power of 2, 1 and 4. Our main object is the eigenvalue distribution of ensembles of Hermitian random matrices of order n, (the power equals 2). We are concerned with the normalization constant and the distribution of a certain linear statistic, namely the sum of the evaluation of functions at the eigenvalues. Ultimately, these generating functions can be characterized by the determinant of Hankel matrices generated by the moments of a certain weight function. Such determinants can be expressed as a product of norms of monic orthogonal polynomials and is intimately related to the recurrence coefficients. By making use of the ladder operators and their associated compatibility conditions for orthogonal polynomials, (which the Applicant has made contributions) it is shown that the Hankel determinant can be expressed as something involving a solution to one of the classical Painleve second order non-linear differential equations.The ladder operator approach has been applied by the Applicant and his collaborators to the Shannon entropy, the outage capacity and the error probability of the multiple-input multiple-output wireless communication systems. The Applicant also study multiple-antenna detection of primary user transmission signals in the cognitive radio networks with electrical engineers and computer scientists people regarding detection of signal out of a noisy background with the tools of multi-variate statistics. The Applicant also collaborates with string theorists on counting problems that show up in the moduli space of super-symmetric QCD, a type of quantum field theory. It is of particular interest and of considerable importance to study the behavior of the generating function for large n, where n is the order of the matrix. For large n, the appropriate consideration is about the "double scale " behavior of the Hankel determinant. In this scheme n, the size of the matrix tends to infinity and a parameter say t, a kind of time variable, tends to 0, in a suitable combination s which is finite. Here we discover that the original finite n, and ``larger” Painleve equation degenerates to a ``smaller” Painleve equations, where the matrix size n has disappeared. This is the universal behavior inherent in random matrix. We obtained the hard-to-come-by constant in an asymptotic expansion. In addition to the Hermitian ensembles, the Applicant is also interest in the expectation value of linear statistics in the orthogonal (power equals 1) and symplectic (power equals 4) ensembles. Here, the exponential moment generating functions are expressed in the form of the determinants of identity plus a scalar operator. Here the scalar operator, roughly speaking, has a rank n and rank one component. Large n behavior is found.

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